Algebra II Semester 2 review Solution Guide

1) $\begin{cases} u_0 = 5 \\ u_n = 2.7 u_{n-1} \end{cases}$

$\boxed{u_n = 5(2.7)^n}$

2) $\begin{cases} u_0 = 3500 \\ u_n = 0.75 u_{n-1} \end{cases}$

$\boxed{u_n = 3500(0.75)^n}$

3)

x	y	
0	10	
1	15	$\frac{15}{10} = 1.5$
2	22.5	$\frac{22.5}{15} = 1.5$
3	33.75	1.5
4	50.63	1.5
5	75.94	1.5

$\boxed{y = 10(1.5)^x}$

4)

x	y	
-1	667	
1	375	←
3	211	
6	89	
10	28	←
12	17	

not increasing by 1

can not use ratio

$\boxed{y = 375(0.75)^{x-1}}$

assume exponential

$y = y_1 b^{x-x_1}$

$y = 375 b^{x-1}$

$28 = 375 b^{10-1}$

$\frac{28}{375} = b^9$

$b = \left(\frac{28}{375}\right)^{1/9} = 0.75$

5) $\sqrt[3]{-1000} = \sqrt[3]{-1}\sqrt[3]{10^3}$

$-1 \quad 1000$

$10 \quad 100$

$10 \quad 10$

$= (-10)$

6) $-(256^{1/4})$

$-\sqrt[4]{256} = -\sqrt[4]{2^8}\sqrt[4]{2^8} =$

$2 \quad 128$

$2 \quad 64$

$2 \quad 32$

$2 \quad 16$

$2 \quad 8$

$2 \quad 4$

$2 \quad 2$

(-4)

7) $\left(\sqrt[4]{16}\right)^2 = 2^2 = (4)$

8) $-(25^{-3/2}) = -\left(\frac{1}{25^{3/2}}\right)$

$= -\frac{1}{(\sqrt{25})^3} = -\frac{1}{5^3} = \left(-\frac{1}{125}\right)$

9) $\frac{70^{1/3}}{14^{1/3}} = \left(\frac{70}{14}\right)^{1/3} = 5^{1/3}$

10) $\frac{\sqrt[4]{4}}{\sqrt[4]{32}} = \sqrt[4]{\frac{4}{32}} = \sqrt[4]{\frac{1}{8}}$

11) $\sqrt[4]{\frac{80}{9}} = \frac{\sqrt[4]{80}}{\sqrt[4]{9}} = \frac{\sqrt[4]{16}\sqrt[4]{5}}{\sqrt[4]{9}} = \frac{2\sqrt[4]{5}}{\sqrt[4]{9}}$

12) $\sqrt[3]{375} + \sqrt[3]{81} = 5\sqrt[3]{3} + 3\sqrt[3]{3} = 8\sqrt[3]{3}$

375 → 5, 75; 75 → 5, 25; 25 → 5, 5

81 → 9, 9; 9 → 3, 3; 3, 3

13) $\sqrt[3]{x} + 10 = 16$

$-10 \quad -10$

$(\sqrt[3]{x})^3 = (6)^3$

$x = 6^3 = \boxed{216}$

14) $(\sqrt{x^2+5})^2 = (x+3)^2$

$x^2 + 5 = x^2 + 6x + 9$

$-x^2 \quad -x^2$

$5 = 6x + 9$

$-9 \quad -9$

$\frac{-4}{6} = \frac{6x}{6}$

$\boxed{-\frac{2}{3} = x}$

+	x	3
x	x^2	$3x$
3	$3x$	9

15) $\frac{3(x+1)^{4/3}}{3} = \frac{48}{3}$

$\left((x+1)^{4/5}\right)^{3/4} = (16)^{3/4}$

$x + 1 = 8$

$-1 \quad -1$

$\boxed{x = 7}$

16) $\sqrt{2x+10} - 2\sqrt{x} = 0$

$(\sqrt{2x+10})^2 = (2\sqrt{x})^2$

$2x + 10 = 4x$

$10 = 2x$

$\boxed{5 = x}$

17-20 | Simplify ⇒ can not solve

17) $(2^3 x^2)^5$

$2^{15} x^{10}$

18) $\frac{x^{11} y^{10}}{x^3 y^{-1}} = x^{11} x^{-3} y^{10} y^{1}$

$= x^{14} y^{11}$

19) $\frac{x^{-1} y}{x y^{-2}} = x^{-1-1} y^{1-2}$

$x^{-2} y^3 = \frac{y^3}{x^2}$

20) $\frac{y^{10}}{2x^3} \cdot \frac{20x^{14}}{xy^6}$

$\frac{20 x^{14} y^{10}}{2 x^3 x y^6} = 10 x^{10} y^4$

21) $f(x) = 3x - 7 \quad g(x) = x^2 - 2x$

$f(f(x)) = f(3x-7)$

$= 3(3x-7) - 7$

$= 9x - 21 - 7$

$f(f(x)) = 9x - 28$

22) $f(g(x)) = f(x^2 - 2x)$

$= 3(x^2 - 2x) - 7$

$f(g(x)) = 3x^2 - 6x - 7$

23) $g(f(x)) = g(3x-7)$

$= (3x-7)^2 - 2(3x-7)$

$= 9x^2 - 42x + 49 - 6x + 14$

$g(f(x)) = 9x^2 - 48x + 63$

24) $g(g(x)) = g(x^2 - 2x)$

$= (x^2 - 2x)^2 - 2(x^2 - 2x)$

$= x^4 - 4x^3 + 4x^2 - 2x^2 + 4x$

$g(g(x)) = x^4 - 4x^3 + 2x^2 + 4x$

25) $f(x) = 3x - 7$

$x = 3y - 7$

$x + 7 = 3y$

$\frac{x+7}{3} = y$

$\boxed{f^{-1}(x) = \frac{1}{3}(x+7)}$

26) $f(x) = x^3 + 2$

$x = y^3 + 2$

$x - 2 = y^3$

$(x-2)^{1/3} = y$

$\boxed{f^{-1}(x) = (x-2)^{1/3}}$

27) $f(x) = 3\log_5(x+1)$

$x = 3\log_5(y+1)$

$\frac{x}{3} = \log_5(y+1)$

$5^{x/3} = y+1$

$5^{x/3} - 1 = y$

$\boxed{f^{-1}(x) = 5^{x/3} - 1}$

28) $f(x) = \frac{1}{2}e^{3x+1}$

$x = \frac{1}{2}e^{3y+1}$

$2x = e^{3y+1}$

$\ln 2x \Leftarrow \log_e 2x = 3y + 1$

$\ln 2x - 1 = 3y$

$\frac{-1 + \ln 2x}{3} = y$

$\boxed{f^{-1}(x) = \frac{1}{3}(-1 + \ln 2x)}$

29) $\log_4 x = -13 \Rightarrow \boxed{4^{-13} = x}$

30) $\log_r z = w \Rightarrow \boxed{r^w = z}$

31) $e^n = 17 \Rightarrow \boxed{\log_e 17 = n \text{ or } \ln 17 = n}$

32) $f^g = g \Rightarrow \boxed{\log_f g = g}$

33) $\log_3 \frac{3x^4}{y^2} = \log_3 3x^4 - \log_3 y^2 = \log_3 3 + \log_3 x^4 - 2\log_3 y$

$= \boxed{\log_3 3 + 4\log_3 x - 2\log_3 y}$

34) $\log_n \sqrt[4]{\frac{y^2}{16x^7}} = \log_n \frac{1}{4}\left(\frac{y^2}{16x^7}\right) = \frac{1}{4}\log_n \frac{y^2}{16x^7}$

$= \frac{1}{4}\left(\log_n y^2 - \log_n 16x^7\right) = \frac{1}{4}\left(2\log_n y - (\log_n 16 + \log_n x^7)\right)$

$= \frac{1}{4}\left(2\log_n y - (\log_n 16 + 7\log_n x)\right)$

$= \frac{1}{2}\log_n y - \frac{1}{4}\log_n 16 - \frac{7}{4}\log_n x$

35) $3\log_5 4 + \frac{1}{2}\log_5 x - \frac{3}{4}\log_5 y = \log_5 4 + \log_5 x^{1/2} - \log_5 y^{3/4}$

$= \log_5 \frac{4x^{1/2}}{y^{3/4}}$

36) $4\log_2 x - (2\log_2 3 + 5\log_2 y)$

$\log_2 x^4 - (\log_2 3^2 + \log_2 y^5)$

$\log_2 x^4 - \log_2 9y^5 = \log_2 \frac{x^4}{9y^5}$

37) $3^{2x+1} = 27^{2x-1}$

$\log_3 3^{2x+1} = \log_3 27^{2x-1}$

$(2x+1)\log_3 3 = (2x-1)\log_3 27$

$1(2x+1) = 3(2x-1)$

$2x+1 = 6x-3$

$4 = 4x$

$1 = x$

38) $4e^{3x-2} = 50$

$e^{3x-2} = \frac{50}{4} = 12.5$

$\ln 12.5 = \log_e 12.5 = 3x-2$

$\ln 12.5 + 2 = 3x$

$x = \frac{\ln 12.5 + 2}{3}$

$x = 1.51$

39) $\log_5 x - 2\log_5 7 = \frac{1}{3}\log_5 27$

$\log_5 x = \log_5 7^2 + \log_5 27^{1/3}$

$\log_5 x = \log_5 (49 \cdot 3)$

$\log_5 x = \log_5 (147)$

$x = 147$

40) $2\ln 4 + 3\ln x = \frac{1}{2}$

$\ln 4^2 + \ln x^3 = \frac{1}{2}$

$\ln 16x^3 = \frac{1}{2}$

$e^{1/2} = 16x^3$

$\frac{e^{1/2}}{16} = x^3$

$\sqrt[3]{\frac{e^{1/2}}{16}} = x$

$x = 0.683$

41) $4000 = 3000\left(1 + \frac{0.035}{12}\right)^{12x}$

$\frac{4}{3} = 1.00292^{12x}$

$\log_{1.00292}(4/3) = 12x$

$98.665 = 12x$

$8.22 = x$

$x =$ 8 years, 3 months

42) $(0, 300)$

$(42, 150)$

$y = 300(b)^x$

$150 = 300(b)^{42}$

$\frac{1}{2} = b^{42}$

$\left(\frac{1}{2}\right)^{1/42} = b$

$b = 0.9836$

$40 = 300(0.9836)^x$

$\frac{40}{300} = 0.9836^x$

$x = \log_{0.9836}\left(\frac{40}{300}\right)$

$x = \frac{\log(40/300)}{\log 0.9836} = 122$

122 hours

43) $y = 19{,}750(1-0.13)^4$

$\approx 11{,}314.73$

44) x = adults
y = students

x	y
2	1
4	2
6	3

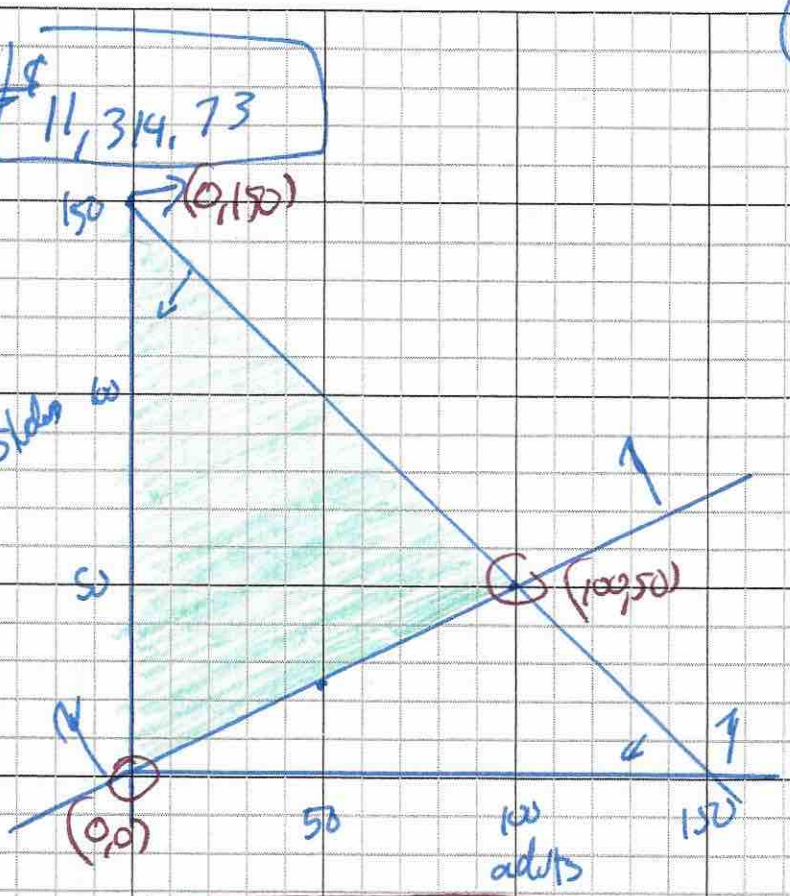
$$\begin{cases} x+y \le 150 \\ x \le 2y \Rightarrow y \ge \frac{1}{2}x \\ x \ge 0 \\ y \ge 0 \end{cases}$$

$P = 6x + 3y$

$(0,0) = 6(0)+3(0) = 0$

$(0,150) = 6(0)+3(150) = 450$

$(100,50) = 6(100)+3(50) = 750$



100 adults & 50 students

45) $y = -\frac{3}{5}(x-4)^2 - 3$

vertex $(4,-3)$
axis of symmetry $x = 4$

roots $0 = -\frac{3}{5}(x-4)^2 - 3$

$-\frac{5}{3}(3) = \left(-\frac{3}{5}(x-4)^2\right)\left(-\frac{5}{3}\right)$

$-15 = (x-4)^2$

no real roots

y-int $y = -\frac{3}{5}(0-4)^2 - 3$

$= -\frac{3}{5}(-4)^2 - 3$

$= -12.6$

$(0,-12.6)$

opens down

46) $f(x) = 3(x+2)(x-6)$

opens up
roots $(-2,0)$ & $(6,0)$

axis of sym $x = \frac{-2+6}{2} = 2$

$x = 2$

vertex $3(2+2)(2-6)$
$3(4)(-4)$
-48

$(2,-48)$

y-int $3(0+2)(0-6)$
$3(2)(-6)$
-36

$(0,-36)$

47) $y = -x^2 + 7x - 12$

opens down

y-int $(0, -12)$

roots $0 = -x^2 + 7x - 12$

$x = \frac{-7 \pm \sqrt{7^2 - 4(-1)(-12)}}{2(-1)}$

$x = \frac{-7 \pm \sqrt{49 - 48}}{-2}$

$x = \frac{-7 \pm 1}{-2}$ $\quad \frac{-6}{-2} = 3$ $\quad \frac{-8}{-2} = 4$

roots $(3, 0)$ & $(4, 0)$

axis of symmetry $x =$

$x = \frac{3+4}{2} = 3.5$

$x = 3.5$

vertex $(3.5, 0.25)$

$-(3.5)^2 + 7(3.5) - 12$

0.25

48) $f(x) = 3x^2 + 4x + 7$

opens up

y-int $(0, 7)$

roots $0 = 3x^2 + 4x + 7$

$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(7)}}{2(3)}$

$x = \frac{-4 \pm \sqrt{-68}}{6}$

no real roots

axis of sym $x = -\frac{2}{3}$

$\frac{-4 + \sqrt{-68}}{6} + \frac{-4 - \sqrt{-68}}{6}$ / 2

$= \frac{1}{2}\left(\frac{-8}{6}\right) = -\frac{4}{6} = \frac{2}{3}$

vertex $\left(-\frac{2}{3}, 5\frac{2}{3}\right)$

$3\left(-\frac{2}{3}\right)^2 + 4\left(-\frac{2}{3}\right) + 7$

5.67

49) 3 up, 5 left $\quad y = (x+5)^2 + 3$

50)

x	y					
0	-2					
		5				
1	3		8			
		13		-12		
2	16		-4		120	
		9		108		-180
3	25		104			
		113		-72		
4	138		32			
		145				
5	283					

Greater than 4th degree

(error in data)

51)

x	y					
0	7					
		+1				
1	8		-4			
		-3		-36		
2	5		-40		-24	
		-43		-60		
3	-38		-100			
		-143		-84		
4	-181		-184			
		-327				
5	-508					

4th degree

$y = -x^4 + 5x^2 - 3x + 7$

52) 3rd degree

$y = a(x+1)(x+1)(x-3)$

$(0, -6)$ $-6 = a(0+1)(0+1)(0-3)$

$-6 = a(-3)$

$2 = a$

$\boxed{y = 2(x+1)(x+1)(x-3)}$

53) 4th degree

$y = a(x+3)(x+1)(x-1)(x-2)$

$(0, -6)$ $-6 = a(0+3)(0+1)(0-1)(0-2)$

$-6 = a(6)$

$-1 = a$

$\boxed{y = -1(x+3)(x+1)(x-1)(x-2)}$

54) $(3+2i) + (5-7i)$

$(3+5) + (2i-7i)$

$\boxed{8-5i}$

55) $(2-3i) - (5-6i)$

$(2-5) + (-3i - -6i)$

$\boxed{-3+3i}$

56) $(3+2i)(4-5i)$

$12-15i+8i-10i^2$

$12-7i+10$

$\boxed{22-7i}$

57) $\dfrac{(5-2i)(3-7i)}{(3+7i)(3-7i)}$

$= \dfrac{15-35i-6i+14i^2}{3^2+7^2} = \dfrac{15-41i-14}{9+49}$

$\boxed{\dfrac{1-41i}{58}}$

58) $x^2+11x+24=0$

$(x+3)(x+8)=0$

$\boxed{x=-3 \text{ or } x=-8}$

59) $4(x+6)^2=160$

$(x+6)^2=40$

$x+6=\pm\sqrt{40}=\pm 2\sqrt{10}$

$\boxed{x=-6\pm 2\sqrt{10}}$

60) $4x^2+6x+7=x^2-4x$

$3x^2+10x+7=0$

$x=\dfrac{-10\pm\sqrt{10^2-4(3)(7)}}{2(3)}$

$x=\dfrac{-10\pm\sqrt{16}}{6}$

$x=\dfrac{-10\pm 4}{6}$ $\boxed{x=-1,\ x=-7/3}$

61) $10x^2+8x-1=0$

$x=\dfrac{-8\pm\sqrt{8^2-4(10)(-1)}}{2(10)}$

$x=\dfrac{-8\pm\sqrt{104}}{20}$

$\boxed{x=\dfrac{-8\pm 2\sqrt{26}}{20}}$

$\boxed{x=\dfrac{-4\pm\sqrt{26}}{10}}$

62) $x^3 - 12x^2 + 12x + 80$ 10 is a zero

	1	−12	12	80
		10	−20	−80
10	1	−2	−8	0

$x^2 - 2x - 8 = (x-4)(x+2)$

$\boxed{(x-10)(x-4)(x+2)}$

63) $f(x) = 16x^4 + 54$

$0 = 16x^4 + 54$

$16x^4 = -54$

$x^4 = -3.375$

no real roots

64) $f(x) = 3x^3 - 2x^2 - 9x + 6$

	3	−2	−9	6
		9	21	36
−3	3	7	12	42
	1	3	4	14
	1	1	2	6
	1	2	3	

Possible roots $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$

	3	−2	−9	6
		2	0	−6
$\frac{2}{3}$	3	0	−9	0

$3x^2 - 9 = 3(x^2 - 3) \pm \sqrt{3}$

roots $\boxed{\frac{2}{3}, \pm\sqrt{3}}$

65) 4 roots (−1, 0) (1, 0) (1, 0) (3, 0)

66) roots (−3, 0) (−1, 0) (0, 0) (1, 0) (3, 0)

67)

$$2x+3 \overline{)\,6x^2+x+7}\quad \Big|\; 3x-4$$

$\frac{6x^2}{2x}=3x$

$3x(2x+3)$

$-(6x^2+9x)$

$-8x+7$

$\frac{-8x}{2x}=-4$

$-4(2x+3)$

$-(-8x-12)$

19

$$3x-4+\frac{19}{2x+3}$$

68)

$$x^3+x^2-5 \overline{)\,2x^4+2x^3-10x-9}\quad \Big|\; 2x$$

$\frac{2x^4}{x^3}=2x$

$-(2x^4+2x^3-10x)$

-9

$$2x-\frac{9}{x^3+x^2-5}$$

69) $(x^3-7x-6)\div(x-2)$

	1	0	−7	−6
		2	4	−6
2	1	2	−3	−12

$$x^2+2x-3-\frac{12}{x-2}$$

70) $(4x^4+5x^3+2x^2-1)\div(x+1)$

	4	5	2	0	−1
		−4	−1	−1	1
−1	4	1	1	−1	0

$$4x^3+x^2+x-1$$