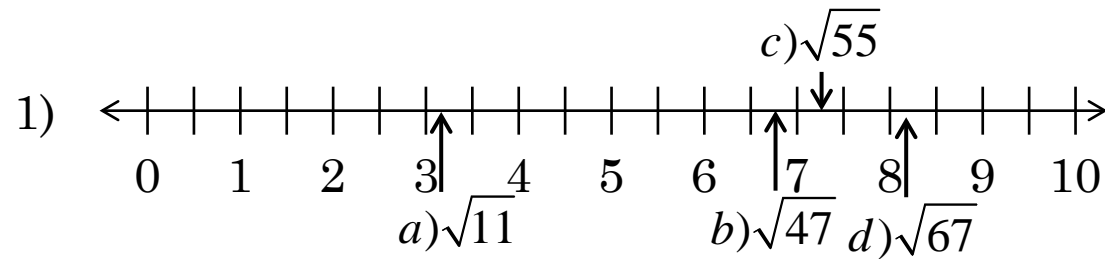


ALGEBRA II

Chapter 5

Exponential, Power and Logarithmic Functions

REFRESH YOUR SKILLS



2a) $2\sqrt{6}$ 2b) $5\sqrt{3}$ 2c) $3\sqrt{5}$ 2d) $2\sqrt{10}$ 2e) $10\sqrt{3}$

3a) *iii, B* 3b) *i, C* 3c) *iv, D* 3d) *ii, A*

4a) $>$ 4b) $<$ 4c) $>$ 4d) $<$



SECTION 5.1A EXPONENTIAL FUNCTIONS

$$1a) f(5) \approx 3.52$$

$$1b) g(14) \approx 19,528.32$$

$$1c) h(24) \approx 22.92$$

$$1d) j(37) \approx 3332.20$$

$$2a) 16, 12, 9; y = 16(0.75)^x$$

$$2b) 24, 36, 54; y = 24(1.5)^x$$

$$3a) f(0) = 125, f(1) = 75, f(2) = 45$$

$$u_0 = 125 \quad u_n = 0.6u_{n-1} \text{ where } n \geq 1$$

$$3b) f(0) = 3, f(1) = 6, f(2) = 12$$

$$u_0 = 3 \quad u_n = 2u_{n-1} \text{ where } n \geq 1$$

$$4a) 0.75; 25\% \text{ decrease}$$

$$4b) 1.\bar{3}; 33.\bar{3}\% \text{ increase}$$

$$4c) 0.94; 6\% \text{ decrease}$$

$$4d) 1.0638; 6.38\% \text{ increase}$$



SECTION 5.1B EXPONENTIAL FUNCTIONS

$$5a) u_0 = 1.211 \quad u_n = 1.015u_{n-1}$$

5b)

5b) y represents the population x years

$$\text{after 1995; } y = 1.211(1.015)^x$$

5c) The equation predicts that the population of China in 2006 was 1.426 billion. This is larger than the actual value, so the population is growing at a slower rate than it was in 1995

Year	Pop. (Billions)
1995	1.211
1996	1.229
1997	1.248
1998	1.266
1999	1.285
2000	1.305
2001	1.324
2002	1.344

$$6a) h = 2.56(2.5)^d ; \text{ day 5} \Rightarrow 250\text{cm}; \text{ day 6} \Rightarrow 625\text{cm}$$

$$6b) h = 2.56(2.5)^{3.5} \approx 63.25$$

$$6c) 728\text{cm}$$

$$6d) 0.76 \text{ day, or 18 hours}$$

$$6e) 11\text{days } 13 \text{ hours, or 9 P.M. on day 11}$$



SECTION 5.1C EXPONENTIAL FUNCTIONS

$$8a) y = 2^{x-3}$$

$$8b) y - 2 = 2^x, \text{ or } y = 2^x + 2$$

$$8c) \frac{y}{3} = 2^x, \text{ or } y = 3 \cdot 2^x$$

$$8d) y = 2^{x/3}$$

$$10a) y - 5 = 0.5^x, \text{ or } y = 5 + 0.5^x$$

$$10b) -(y - 5) = 0.5^x, \text{ or } y = -0.5^x + 5$$

$$10c) y + 2 = 0.5^{x-1}, \text{ or } y = -2 + 0.5^{x-1}$$

$$10d) \frac{y}{3} = 0.5^{x/2}, \text{ or } y = 3(0.5)^{x/2}$$

$$15a) A = 5000(1 + 0.035)^5$$

$$15b) S = 5200(1 + 0.032)^5$$

15c) After 14 years, Austin will have \$8,093.47,
and Sami will have \$8,082



SECTION 5.2A PROPERTIES OF EXP. FUNCTIONS

1a) $\frac{1}{125}$ 1b) -36 1c) $-\frac{1}{81}$ 1d) $\frac{1}{144}$ 1e) $\frac{16}{9}$ 1f) $\frac{7}{2}$

2a) a^5 2b) b^4 2c) c^{20} 2d) e^3

3a) false - different bases

3b) false - order of operation

3c) false - order of operations

3d) true

4a) $x = -2$

4b) $x = -3$

4c) $x = -5$

4d) $x = 0$

5a) $x \approx 3.27$

5b) $x = 784$

5c) $x \approx 0.16$

5d) $x \approx 0.50$

5e) $x \approx 1.07$

5f) $x = 1$

6a) x^{12}

6b) $8x^{12}$

6c) $10x^7$

6d) $12x^5$

6e) $8x^{12}$

6f) $\frac{1}{25}x^{-12}$

SECTION 5.2B PROPERTIES OF EXP. FUNCTIONS

7) Must show work and describe findings

$$10a) y - 4 = x^3, \text{ or } y = x^3 + 4$$

$$10b) y = (x + 2)^3 \quad 10c) 4y = x^3, \text{ or } y = \frac{1}{4}x^3$$

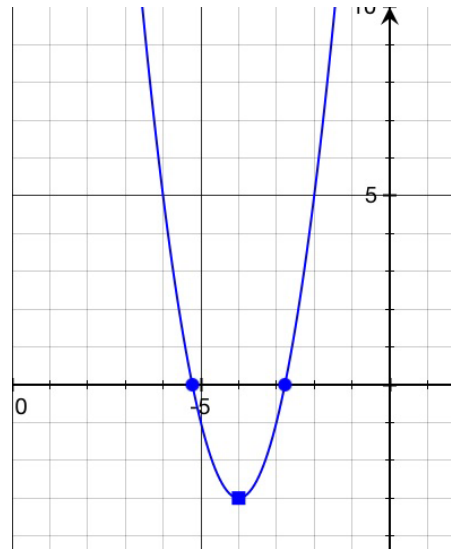
$$10d) 8(y + 2) = x^2, \text{ or } y + 2 = \frac{1}{8}x^2, \text{ or } y = \frac{1}{8}x^2 - 2$$

$$13a) x = 7$$

$$13b) x = -\frac{1}{2}$$

$$13c) x = 0$$

$$16a) y = 2(x + 4)^2 - 3$$



SECTION 5.3A RATIONAL EXP. AND ROOTS

1 $(a, e, j); (b, d, g); (c, i); (f, h)$

2) must give reasons

2a) power

2b) power

2c) exponential

2d) power

2e) power

2f) power

2g) exponential

2h) power

2i) power

2j) neither

2k) power

2l) exponential

3a) $a^{1/6}$

3b) $b^{4/5}$

3c) $c^{-1/2}$

3d) $d^{7/5}$

4) must explain steps

4a) $a = 4.2^6 \approx 5489.032$

4b) $b = 14.3^{5/4} \approx 27.808$

4c) $c = 0.55^{-2} \approx 3.306$

4d) $d = 23^{5/7} \approx 9.390$

5) 490 W/cm²

6) must show all work and explain your answer for part g



SECTION 5.3B RATIONAL EXP. AND ROOTS

9a) exponential 9b) neither 9c) exponential 9d) power

$$10a) y = 3 + (x - 2)^{3/4}$$

$$10b) y = 1 + [-(x - 5)]^{3/4}$$

$$10c) y = 4 + \left(\frac{x}{4}\right)^{3/4}$$

$$10d) y = 4 \left(\frac{x - 3}{2}\right)^{3/4}$$

$$11a) x = \left(\frac{13}{9}\right)^5 \approx 6.29 \quad 11b) 180^{1/4} \approx 3.66 \quad 11c) x = \left(\frac{\sqrt{35}}{4}\right)^{3/2} \approx 1.80$$

13a) work 13b) $k = 492$ 13c) 8.2L 13d) 32.8 mm Hg

14a) $27x^9$ 14b) $16x^9$ 14c) $0.2x^{-1}$ 14d) $108x^8$ 14e) $18x^2y^4$

15a) $y = (x + 4)^2$ 15b) $y = x^2 + 1$ 15c) $y = -(x + 5)^2 + 2$

15d) $y = (x - 3)^2 - 4$ 15e) $y = \sqrt{x + 3}$ 15f) $y = \sqrt{x} - 1$

15g) $y = \sqrt{x + 2} + 1$ 15h) $y = -\sqrt{x - 1} - 1$



SECTION 5.4A APPLICATIONS

1a) $x = 50^{1/5} \approx 2.187$

1b) $x = 29.791$

1c) *no real solution*

2a) $x = 625$

2b) $x = 1$

2c) $x = 512$

2d) $x \approx 0.951$

2e) $x \approx 0.456$

3a) $9x^4$

3b) $8x^6$

3c) $216x^{-18}$

4a) $100r^6$

4b) $100r^6 = 50$

4c) $r = 0.891; 89.1\%$

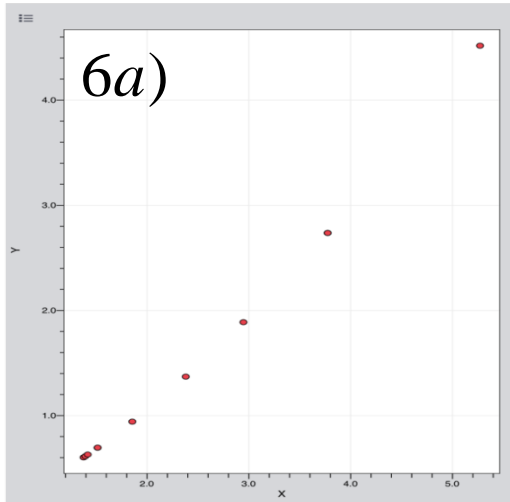
5a) She must replace y with $y-7$ and y_1 with $y_1 - 7$; $y - y = (y_1 - 7) \cdot b^{x-x_1}$

5b) $y - 7 = (105 - 7)b^{x-1}; \left(\frac{y-7}{98}\right)^{1/(x-1)} = b$

5c & d) Answers will vary



SECTION 5.4B APPLICATIONS



6b) Sample $\hat{y} = 0.37x^{1.5}$

6c) $\approx 1,229,200\text{km}$

6d) $545.390d$

11a) 0.0466 , or 4.66% per year

11b) $6.6g$

11c) $y = 6.6(1 - 0.0466)^x = 6.6(0.9534)^x$

11d) $0.6g$

11e) 14.5 yr



SECTION 5.5A BUILDING INVERSES

1) $(-3, -2), (-1, 0), (2, 2), (6, 4)$

2a) $g(2) = 9$

2a) $g^{-1}(9) = 2$

2c) $g^{-1}(20) = \frac{15}{2}$

3) c ; need explanation

4) a & e ; b & d ; f & h

5a) $f(7) = 4$; $g(4) = 7$

5b) They might be inverses

5c) $f(1) = -2$; $g(-2) = 5$ 5d) They are not inverse functions, at least not over the entire domain and range.

5e) $f(x)$ for $x \geq 3$; $g(x)$ for $x \geq -4$

6a) $x = 34$

6b) $f^{-1}(x) = 2 + (x - 4)^{5/3}$

6c) must explain

10a) (*med - med line*) $f(x) \approx -0.006546x + 14.75$

10b) $f^{-1}(x) \approx -152.76x + 2252.76$

10c) (*med - med line*) $g(x) = -0.003545x + 58.81$

10d) $g^{-1}(x) \approx -282.1x + 16591$

10e) $\approx -14.44^\circ F$



SECTION 5.5B BUILDING INVERSES

12) *Score is 1; correct problems 2 – 4*

$$14a) c(x) = 7.18 + 3.98x$$

$$14b) \$39.02$$

$$14c) g(x) = \frac{x - 7.18}{3.98}$$

$$14d) 12,000 \text{ gal}$$

14e) *show work*

$$14f) \approx \$6$$

14g) Answers will vary, volume $\approx 346,000 \text{ in}^3$

$$17a) x = \frac{9}{2}$$

$$17b) x = -\frac{1}{2}$$

$$17c) x = 1$$

$$16) f(x) = 12.6(1.5)^{x-2} \text{ or } f(x) = 42.525(1.5)^{x-5}$$



SECTION 5.6 LOGARITHMS

$$1a) 10^x = 1000$$

$$1b) 5^x = 625$$

$$1c) 7^{1/2} = x$$

$$1d) x^3 = 8$$

$$1e) 5^{-2} = x$$

$$1f) 6^x = 1$$

$$2a) x = 3$$

$$2b) x = 4$$

$$2c) x = \sqrt{7} \approx 2.65$$

$$2d) x = 2$$

$$2e) x = \frac{1}{25}$$

$$2f) x = 0$$

8b) 6025 yr ago. Assumes carbon-14 concentration has not changed.

$$3a) x = \log_{10} 0.001; x = -3$$

$$3b) x = \log_5 100; x \approx 2.8614$$

$$3c) x = \log_{35} 8; x \approx 0.5849$$

$$3d) x = \log_{0.4} 5; x \approx -1.7565$$

$$3e) x = \log_{0.8} 0.03; x \approx 15.7144$$

$$3f) x = \log_{17} 0.5; x \approx -0.2447$$

$$5a) \text{false}; x = \log_6 12$$

$$5b) \text{false}; 2^x = 5$$

$$5c) \text{false}; x = \frac{\log 5.5}{\log 3}$$

$$5d) \text{false}; x = \log_3 7$$

6) approximately 25 min

$$8a) y = 100(0.999879)^x$$

$$10a) x = 345$$

$$10b) x \approx 2.25$$

$$12a) C_1 = 32.7,$$

$$C_2 = 65.4,$$

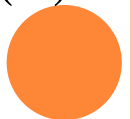
$$C_3 = 130.8,$$

$$C_6 = 1046.4,$$

$$C_7 = 2092.8,$$

$$C_8 = 4185.6$$

$$12b) y = 16.35(2)^x$$



SECTION 5.7A PROPERTIES OF LOGARITHMS

1a) $\log 55$ 1b) $\log 8$ 1c) $\log 4$ 1d) $\log \frac{1}{36}$ 1e) $\log 63$

2a) $\log 2 + \log 11$ 2b) *ex.* $\log 26 - \log 2$ 2c) $\log 3 + \log 13$

2d) *ex.* $\log 14 - \log 2$

3a) $x \log 5$ 3b) $2 \log x$ 3c) $\frac{1}{2} \log 3$ 3d) $2x \log 7$

4a) *true* 4b) *false; ex.* $= \log 15$ 4c) *true* 4d) *true*

4e) *false; ex.* $= \log 3$ 4f) *false; ex.* $= \frac{1}{2} \log 7$ 4g) *false; ex.* $\log 7^5 =$

4h) *true* 4i) *false; ex.* $= \log_4 3$ 4j) *true*

5a) $g^h \cdot g^k$; product prop. of exp. 5b) $\log st$; product prop. of log.

5c) f^{w-v} ; quotient prop. of exp. 5d) $\log h - \log k$; quotient prop. of exp.

5e) j^{st} ; power prop. of exp. 5f) $g \log b$; power prop. of log.

5g) $k^{m/n}$; def. of rational exp. 5h) $\log_u t$; Change-of-base

5i) w^{t+s} ; product prop. of exp. 5j) $1/p^h$; def. of negative exp.



SECTION 5.7B PROPERTIES OF LOGARITHMS

$$6a) y = 100(0.999879)^x$$

$$6b) 11,460 \text{ yr}$$

$$6c) \approx 1910 \text{ BC}$$

6d) $y \approx 0$; virtually nothing to measure

$$7a) y = 261.6(2^{x/12})$$

$$7b) 277.2; 293.6; 293.6; 311.1; 329.6; 349.2 \\ 370.0; 392.0; 415.3; 440.0; 466.1; 493.8$$

$$8a) x \approx 3.3816$$

$$8b) x \approx 11.495$$

$$8c) x \approx 11.174$$

$$8d) x \approx 42.739$$

$$10a) 96.5\%$$

$$10b) y = 100(0.965)^x$$

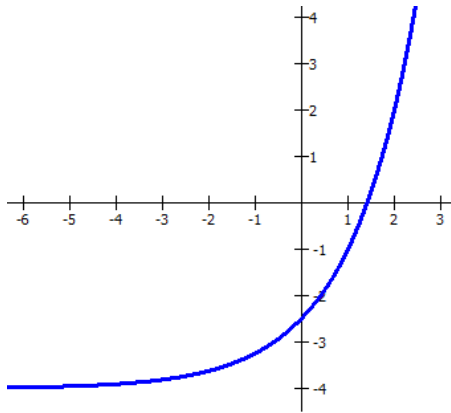
$$10c) 19.465 \text{ min}$$

10d) after one day it is almost all gone

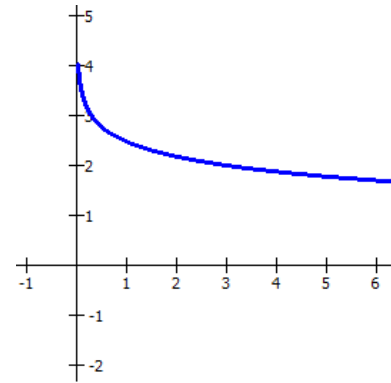


SECTION 5.7C PROPERTIES OF LOGARITHMS

13a) Translated 4 down & 1 right
Dilated 3x Vertical



13b) Translated 2 up
Dilated 3x Horizontal
Reflected across x-axis



$$15a) c = 14h + 20$$

$$\text{Domain : } h > 0$$

$$\text{Range : } c > 20$$

$$15b) a(c(h)) = 1.15c(h) + 25$$

$$\text{Domain : } c(h) > 20$$

$$\text{Range : } a > 48$$

$$15c) a = 16.1h + 48$$



SECTION 5.8A APPLICATIONS OF LOGARITHMS

1a) 2.90309 1b) 11 1c) -4 1d) 1.4123 1e) 2.9303 1f) 5.3246

2) Show your work 3) 16 yr, 4 mo

4a) $h = 146(0.9331226)^{T-4}$

4b) 24.1 h @ 30°C; 63.6 h @ 16°C

4c) 3.9°C

4e) $0^\circ\text{C} \leq T \leq 100^\circ\text{C}$;

Boiling and Freezing point

5a) $f(20) \approx 133.28$ After 20 days,

133 games have been sold.

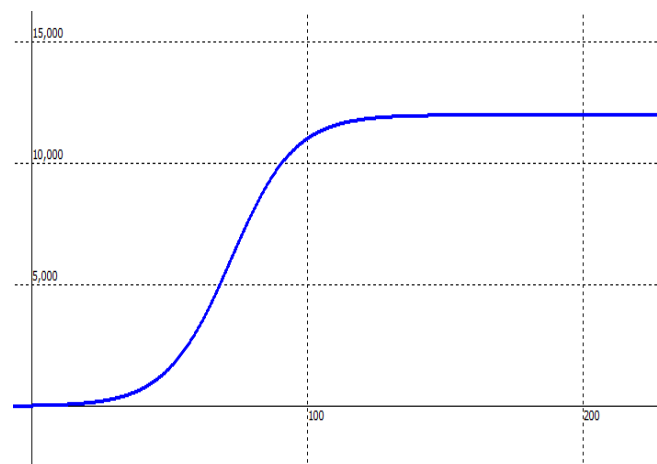
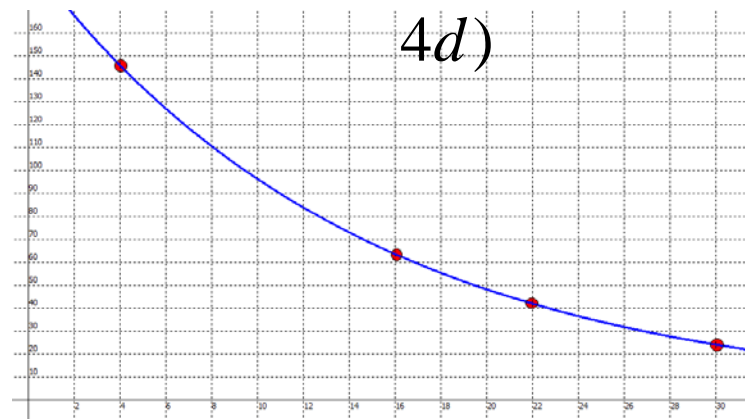
5b) $f(80) \approx 7969.17$ After 80 days,

7969 games have been sold.

5c) $x \approx 72.09$ After 72 days,

6000 games have been sold.

5d) Show Work

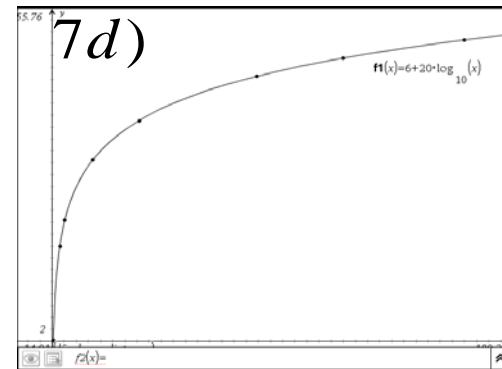
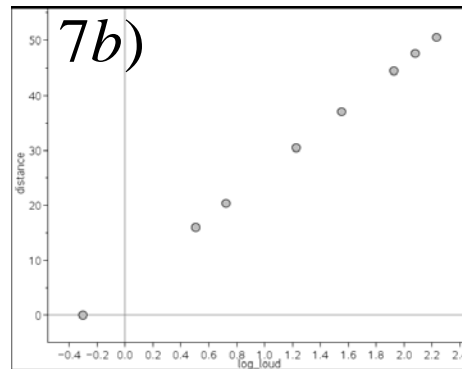
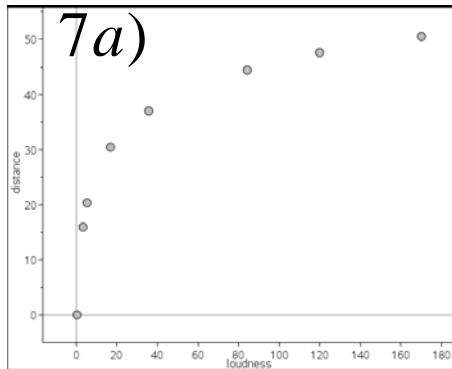


5e) Description

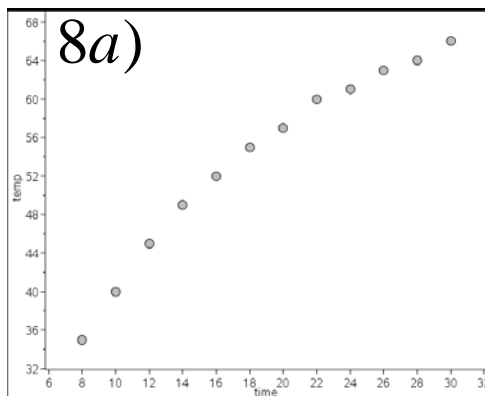


SECTION 5.8B APPLICATIONS OF LOGARITHMS

6a) 30 dB 6b) 65 dB 6c) $5.01 \times 10^{-6} \text{ W/cm}^2$ 6d) 3.6 times as loud



7c) $y = 6 + 20 \log x$



8b) $y = 74 - 66.52(0.9338)^x$

14a) 5.09

14b) $y = 9.1$ or 1.1

14c) 1.40

