

# Algebra II Semester 1 Final Review

1)  $a_1 = 20$   $20, 27, 34, 41, 48 \dots$   
 $a_n = a_{n-1} + 7$

2)  $b_1 = 72$   $72, 162, 364.5, 822.125, 1845.28 \dots$   
 $b_n = 2.25 b_{n-1}$

3)  $c_1 = 100$   $100, 125, 143.75, 157.81, 168.36 \dots$   
 $c_n = 0.75 c_{n-1} + 25$

4)  $15.6, 19.4, 23.2, 27, 30.8, 34.6 \dots$

$19.4 - 15.6 = 3.8$   
 $23.2 - 19.4 = 3.8$   
 $30.8 - 27 = 3.8$  } arithmetic  $u_1 = 15.6$   
 $u_n = u_{n-1} + 3.8, n \geq 2$

5)  $3.2, 10.88, 36.99, 125.77, 427.63 \dots$

$10.88 - 3.2 = 7.68$   $\frac{10.88}{3.2} = 3.4$   
 $36.99 - 10.88 = 26.11$   $\frac{36.99}{10.88} = 3.3998 \approx 3.4$   $\frac{427.63}{125.77} = 3.4$   
 not arithmetic

Geometric

$u_1 = 3.2$   
 $u_n = 3.4 u_{n-1}, n \geq 2$

6)  $200, -140, 180.5, -171.5, 162.9 \dots$  alternates signs - cannot be arithmetic

$\frac{-140}{200} = -0.95$   $\frac{180.5}{-140} = -1.29 \approx -0.95$   
 $\frac{180.5}{-140} = -1.29$   $\frac{162.9}{180.5} = -0.908 \approx -0.95$   
 $u_1 = 200$   
 $u_n = -0.95 u_{n-1}, n \geq 2$

$$7) 1372, 1319.7, 1267.4, 1215.1, \dots$$

$$1372 - 1319.7 = -52.3$$

$$1319.7 - 1267.4 = -52.3$$

$$1267.4 - 1215.1 = -52.3$$

arithmetic

$$u_1 = 1372$$

$$u_n = u_{n-1} - 52.3, n \geq 2$$

$$8) a) \text{ Start } 40 \quad 35\% \text{ growth}$$

$$u_0 \quad \rightarrow (1 + 0.35)$$

$$u_0 = 40$$

$$n \geq 1$$

$$u_n = u_{n-1} (1 + 0.35) \quad \text{or} \quad u_0 = 40 \quad n \geq 1$$

$$u_n = 1.35 u_{n-1}$$

$$b) \text{ Start } 7000 \quad 12\% \text{ decay}$$

$$u_0 \quad (1 - 0.12)$$

$$u_0 = 7000$$

$$n \geq 1$$

$$u_n = u_{n-1} (1 - 0.12) \quad \text{or} \quad u_0 = 7000 \quad n \geq 1$$

$$u_n = 0.88 u_{n-1}$$

$$c) \text{ Deposit } \$500 \quad 0.75\% \text{ interest compounded Qtr}$$

$$u_0$$

$$\text{Qtr} \Rightarrow \frac{0.0075}{4} = 0.001875$$

$$u_0 = 500$$

$$(1 + 0.001875)$$

$$u_n = 1.001875 u_{n-1}, \quad n \geq 1, \quad n = \# \text{ of Quarters}$$

$$d) \text{ starts } 300, \text{ loses } 50, \text{ grow by } 12.5\%$$

$$u_0$$

$$(1 + 0.125)$$

$$u_0 = 300$$

$$u_n = 1.125 u_{n-1} + 50, \quad n \geq 1$$

9) cost \$15,000, depreciates @ 20%  $(1-0.20)$

a)  $u_0 = 15,000$   
 $u_n = 0.8 u_{n-1}, n \geq 1$

b) year	value (\$)	c) year	value \$
0	15,000	6	3932.2
1	12,000	7	3145.7
2	9600	8	2516.6
3	7680	9	2013.3
4	6144	10	1610.6
5	4915.2		

She needs to sell the car  
~~after~~ before 10 years  
 (can keep it for 9 years)

10) loan amount \$17,000, 1.9% compounded monthly  
 monthly payment \$297  $(\frac{0.019}{12})$  each month

a)  $u_0 = 17000$   
 $u_n = u_{n-1} (1 + \frac{0.019}{12}) - 297, n \geq 1, n = \# \text{ of months}$

b) year	n	owe ( $u_n$ )
0	0	17000
1	12	13731
2	24	10399
3	36	7002.7
4	48	3541.7
5	60	14.39

c) after 2 years  
 Value = 9600  
 owes = 10399  
 $- 9600$   
 $- 799$

She still owes \$799, she  
 has nothing to put towards  
 a new car.

d) three years value = 7680  $.70(7680) = 5376$   
 owes 7002.7  $- 7002.7$   
 she still owes \$1626.70 on the old car  $- 1626.7$

$$10) \quad a_1 = 5000 \\ a_n = 0.75a_{n-1} + 2000$$

$$x = 0.75x + 2000 \\ -0.75x - 0.75x$$

$$\frac{0.25x}{0.25} = \frac{2000}{0.25}$$

$$x = 8000$$

$$11) \quad a_1 = 40 \\ a_n = 1.4a_{n-1} + 10$$

$$x = 1.4x + 10 \\ -1.4x - 1.4x$$

$$-0.4x = -10 \\ \frac{-0.4x}{-0.4} = \frac{-10}{-0.4}$$

$$x = 25$$

sequence is increasing  
will not reach 25  
no long term value

$$12) \quad a_1 = 20 \\ a_n = 0.7a_{n-1} + 60$$

$$x = 0.7x + 60 \\ -0.7x - 0.7x$$

$$\frac{0.3x}{0.3} = \frac{60}{0.3}$$

$$x = 200$$

## Unit 2 - Linear Models

$$13) \quad u_0 = 5 \\ u_n = u_{n-1} + 5$$

$$u_n = 5 + 5n$$

$$14) \quad u_0 = -5 \\ u_n = u_{n-1} + 2.5$$

$$u_n = -5 + 2.5n$$

$$15) \quad u_0 = 25 \\ u_n = u_{n-1} - 8.25$$

$$u_n = 25 - 8.25n$$

x	y
0	-8.4
1	-6.1
2	-3.8
3	-1.5
4	8
5	3.1
6	5.4

Recursive

$$u_0 = -8.4$$

$$u_n = u_{n-1} + 2.3$$

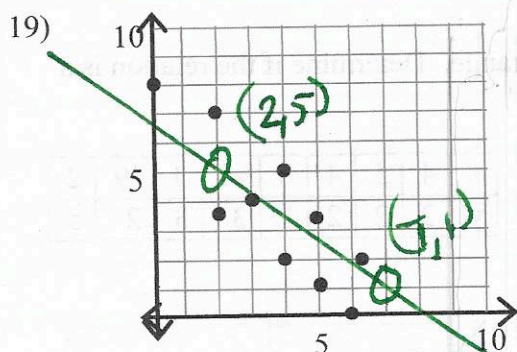
Explicit

$$u_n = -8.4 + 2.3n$$

5

17)  $\begin{cases} (0, 8) \\ (1, 6.5) \\ (2, 5) \\ (3, 2.5) \\ (4, 2) \\ (5, 0.5) \end{cases} \rightarrow -1.5$  recursive  
 $u_0 = 8$   
 $u_n = u_{n-1} - 1.5$   
 explicit  
 $u_n = 8 - 1.5n$

18)  $\begin{cases} (0, 5) \\ (1, 5\frac{1}{3}) \\ (2, 5\frac{2}{3}) \\ (3, 6) \\ (3, 6\frac{1}{3}) \\ (3, 6\frac{2}{3}) \end{cases}$  recursive  
 $u_0 = 5$   
 $u_n = u_{n-1} + \frac{1}{3}$   
 explicit  
 $u_n = 5 + \frac{1}{3}n$



$$\text{slope} = \frac{5-1}{2-7} = -\frac{4}{5}$$

$$y = 5 - \frac{4}{5}(x-2) \quad \text{a} \quad y = 1 - \frac{4}{5}(x-7)$$

21)  $\hat{y} = 0.254x - 491.2$

22) Each year, the enrollment increases by about 0.25 million students or 250,000 students

23) a)  $\hat{y} = 0.254(1975) = 491.2 = 10.45$  million

b)  $\hat{y} = 0.254(1983) = 491.2 = 12.482$  million

c)  $\hat{y} = 0.254(1997) = 491.2 = 16.038$  million

d)  $\hat{y} = 0.254(2011) = 491.2 = 19.594$  million

24)  $\text{RMSE} = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}} = \sqrt{\frac{23.72}{26-2}} = 0.994$

## Unit 3 - Functions &amp; Transformations

25) Domain =  $\{1, 3, 4, 5, 6, 7, 9\}$   
 Range =  $\{2, 3, 5\}$   
 is a function

26) unnumbered Domain  $\{-9, -4, 2, 3, 4, 5, 6\}$   
 Range  $\{2, 3, 5\}$   
 is a function

26) Domain  $\mathbb{R}$   
 Range  $y \geq -2.2$   
 is a function

27) Domain  $x \leq -3$  or  $x \geq 3$   
 Range  $\mathbb{R}$   
 not a function

28)  $g(-3) = -2\sqrt{-3+7} + 5$   
 $= -2\sqrt{4} + 5$   
 $= -2(2) + 5$   
 $= -4 + 5$

$g(-3) = 1$

29)  $f(6) = 3(6-4)^2 - 5$   
 $= 3(2)^2 - 5$   
 $= 3(4) - 5$   
 $= 12 - 5$

$f(6) = 7$

30)  $r(-2) = 2.7$

31)  $h(4) = \frac{2}{3}|4-6|$   
 $= \frac{2}{3}|2|$   
 $= \frac{2}{3}(2)$

$h(4) = 4/3$

32)  $g(8) \cdot h(2)$   
 $(-2\sqrt{8+7} + 5)(\frac{2}{3}|2-6|)$   
 $(-2\sqrt{15} + 5)(\frac{2}{3}|-4|)$   
 $(-2.75)(\frac{2}{3}(4))$

$= -7.32$

33)  $r(-8) + f(-5)$   
 $(-5) + 3(-5-4)^2 - 5$   
 $-5 + 3(-9)^2 - 5$   
 $-5 + 3(81) - 5$   
 $-5 + 243 - 5$

$233$

34)  $2r(0) + 3(g(0))$

$r(0) = -1$

$g(0) = -2\sqrt{0+7} + 5$   
 $= -0.292$

$2(-1) + 3(-0.292)$   
 $= -2.873$

35)  $h(-3) - 5f(3)$

$h(3) = \frac{2}{3}|x-6|$

$3 - 5(-2) = 3 + 10$

$= \frac{2}{3}|3-6| = \frac{2}{3}|-3|$   
 $= \frac{2}{3}(3)$

13

$h(3) = 2$   
 $f(3) = 3(3-4)^2 - 5 = 3(-1)^2 - 5 = -2$

36)  $g(r(6))$

$r(6) = 0.8$

$g(0.8) = -2\sqrt{0.8+7} + 5$   
 $= -2\sqrt{7.8} + 5$

$= -0.586$

37)  $h(-g(-6))$

$g(-6) = -2\sqrt{-6+7} + 5$

$= -2\sqrt{1} + 5 = 3$

$h(-3) = \frac{2}{3}|-3-6|$

$= \frac{2}{3}|-9| = \frac{2}{3}(9) = 6$

38)

$g(x) = 5x - 12$

$t(x) = 4x^2 + 2$

$g(t(x)) = g(4x^2 + 2)$

$= 5(4x^2 + 2) - 12$

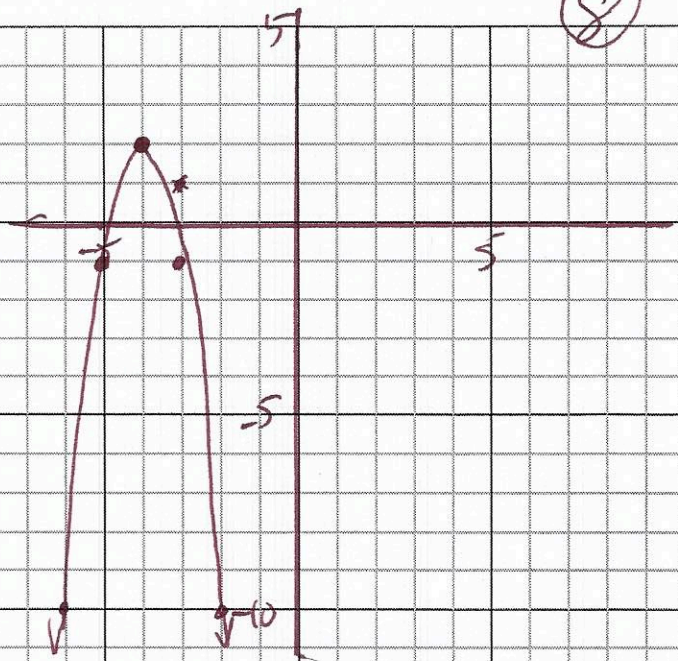
$= 20x^2 + 10 - 12$

$= 20x^2 - 2$

39)  $y = -3(x+4)^2 + 2$

parent  $y = x^2$   
translation 4 left  
vertical reflection 2 up  
vertical Dilaton factor 3

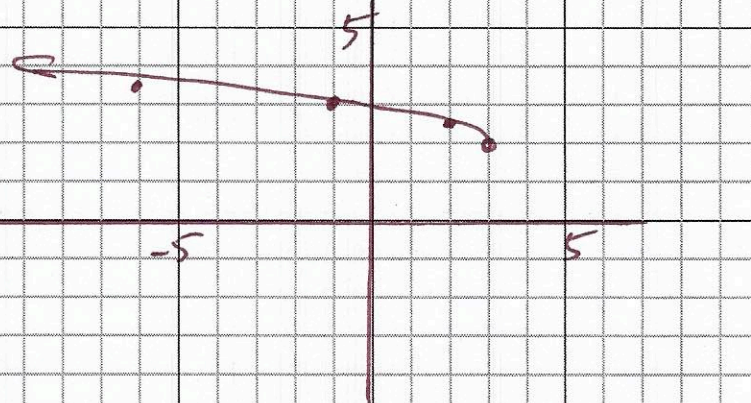
x	y
-3	-1
-2	-10



40)  $y = \frac{1}{2}\sqrt{-(x-3)} + 2$

parent  $y = \sqrt{x}$   
translation 3 right  
2 up  
horizontal reflection  
vertical Dilaton factor  $\frac{1}{2}$

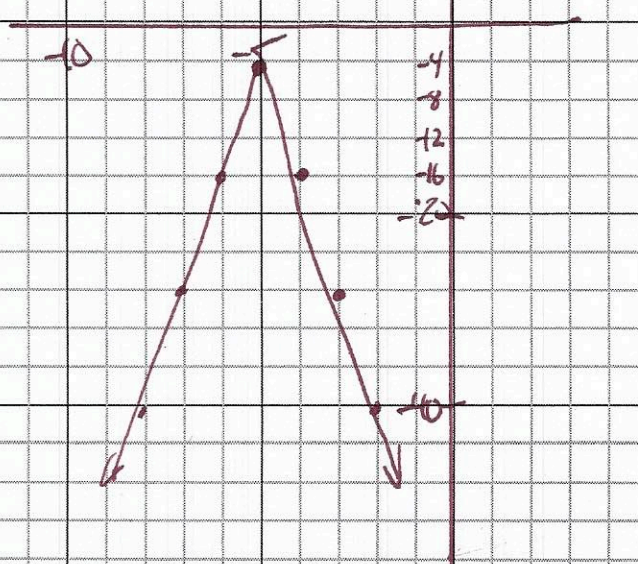
x	y
2	2.5
-1	3
-6	3.5



41)  $y = -4|3(x+5)| - 4$

parent  $y = |x|$   
translation left 5  
down 4  
vertical reflection  
vertical Dilaton factor 4  
horizontal Dilaton factor  $\frac{1}{3}$   
or  
vertical Dilaton factor 12

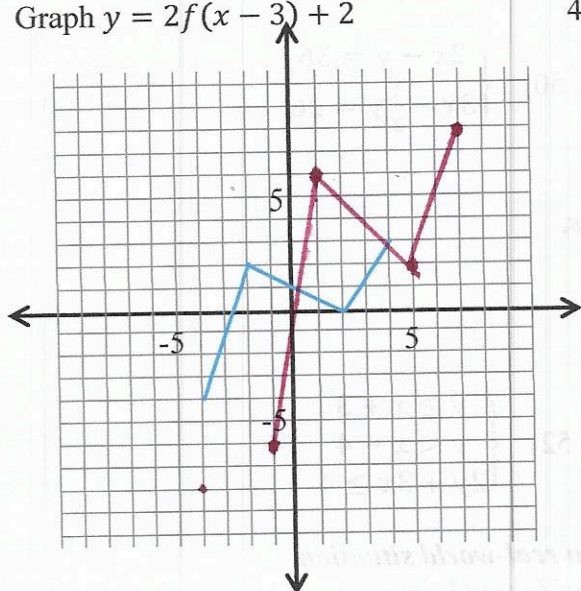
x	y
-4	-16
-3	-28
-2	-40



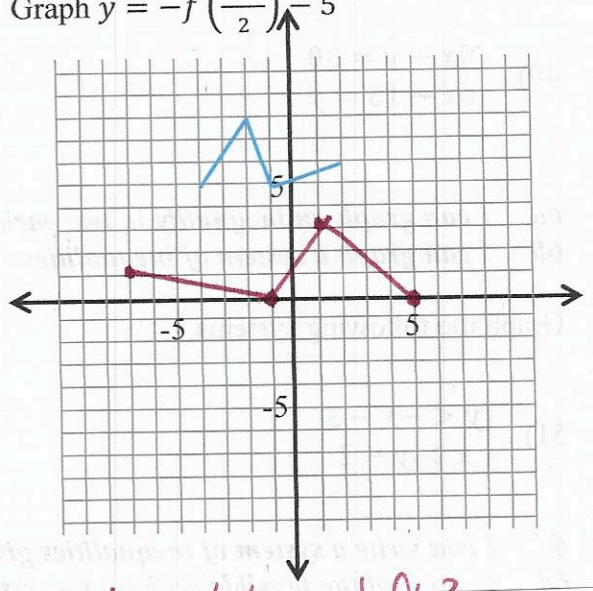


Given  $y = f(x)$  graphed below, graph the transformed function.

42) Graph  $y = 2f(x - 3) + 2$



43) Graph  $y = -f\left(\frac{x+3}{2}\right) - 5$



translation right 3  
up 2  
vert dilation factor 2

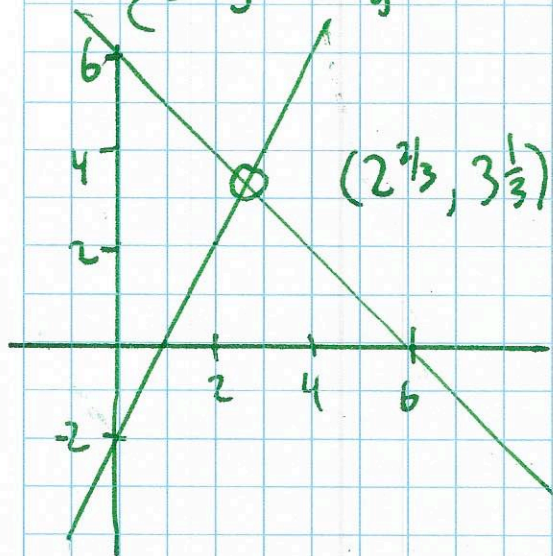
translation left 3  
down 5  
vert reflection  
horizontal dilation factor 2

44) clearance price 50% off  $C(x) = 0.5x$   
 sale price 35% off  $S(x) = \cancel{0.65} (1 - 0.3)x = 0.7x$

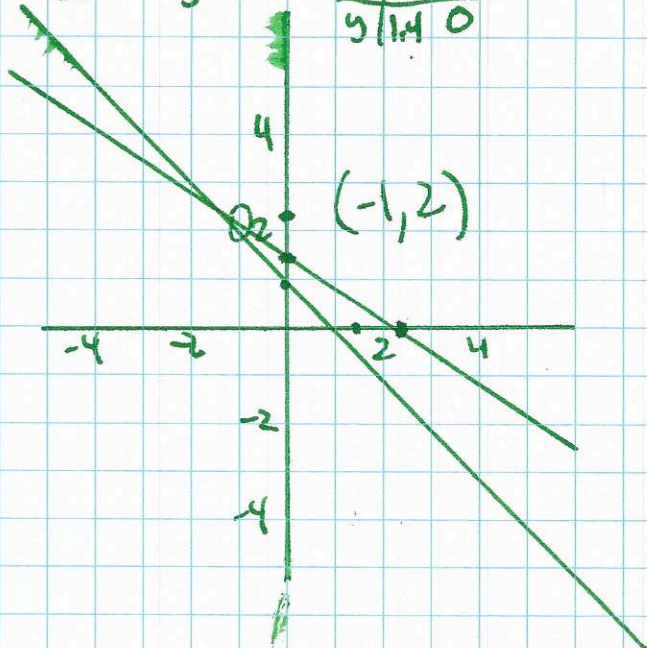
a) \$120 on clearance = \$60  
 \$60 on sale =  $0.7(60) = \text{\$42}$

b)  $S(C(x)) = S(0.5x) = 0.7(0.5x) = 0.35x$   
 $0.35 = 1 - 0.65$   
 you are getting 65% of regular price

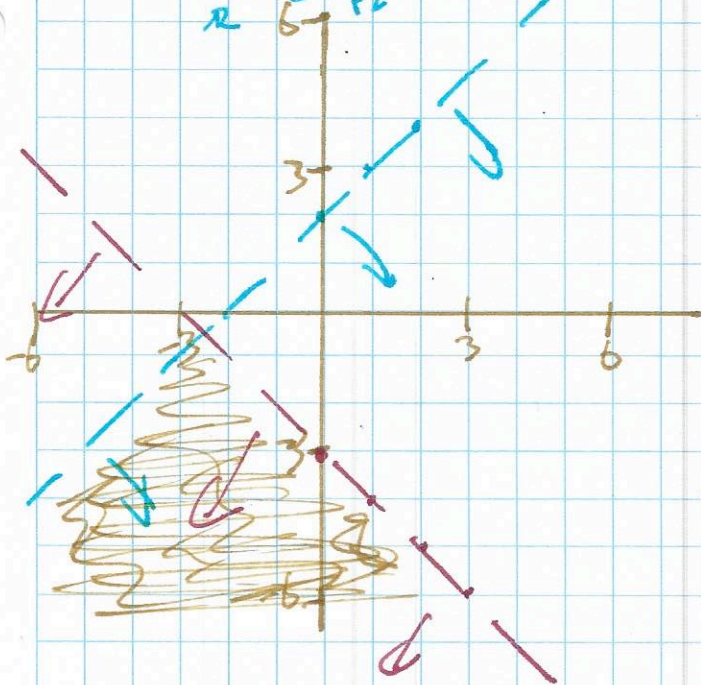
45)  $\begin{cases} x+y=6 \\ 2x-y=2 \end{cases}$   
 $y=6-x$   
 $y=2x-2$



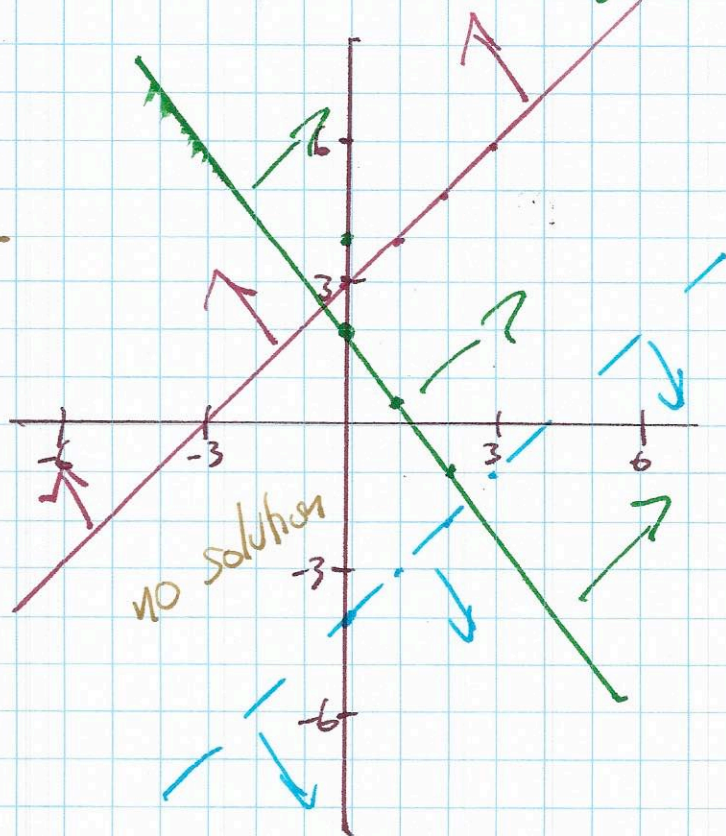
46)  $\begin{cases} x+y=1 \\ 3x+5y=7 \end{cases}$   
 $y=1-x$   
 $\begin{array}{r|rr} x & 1 & 2.33 \\ y & 1.4 & 0 \end{array}$



51)  $y < -x-3$   
 $x > y-2$   
 $x+2 > y$



52)  $y \geq x+3$   
 $y < x-4$   
 $2y+3x \geq 4$   
 $2y \geq 4-3x$   
 $y \geq 2-1.5x$



47)  $6x + 4y = 80$   
 $x - 7y = -2$   
 $\quad +7y \quad +7y$   
 $x = 7y - 2$   
(12, 2)

$6(7y - 2) + 4y = 80$   
 $42y - 12 + 4y = 80$   
 $\frac{46y}{46} = \frac{92}{46}$   
 $y = 2$

$x = 7(2) - 2 = 14 - 2 = 12$

$6(12) + 4(2) = 72 + 8 = 80 \checkmark$

48)  ~~$4x + 7y = 1$~~   $x = 2 - y$   
 ~~$3x + 5y = 7$~~   $x - 2y = 0$

$2 - y - 2y = 0$   
 $2 - 3y = 0$   
 $2 = 3y$   
 $y = \frac{2}{3}$   
( $\frac{1}{3}, \frac{2}{3}$ )

$x = 2 - \frac{2}{3} = \frac{1}{3}$

$\frac{1}{3} - 2(\frac{2}{3}) = \frac{1}{3} - \frac{4}{3} = -1 \checkmark$

49)  $9x - y = 30$   
 $6x - 15 = y \Rightarrow -15 = y - 6x$

$9x - y = 30$   
 $-6x + y = -15$   
 $\frac{3x}{3} = \frac{15}{3}$   
 $x = 5$   
(5, 15)

$6(5) - 15 = y$   
 $30 - 15 = y$   
 $y = 15$

$9(5) - 15 = 45 - 15 = 30 \checkmark$

50)  $2x - y = 36$   
 $-2(3x - \frac{1}{2}y) = 26(2)$

$2x - y = 36$   
 $-6x + y = -52$   
 $\frac{-4x}{-4} = \frac{-16}{-4}$   
 $x = 4$   
(4, -28)

$2(4) - y = 36$   
 $8 - y = 36$   
 $-y = 28$   
 $y = -28$

$3(4) - \frac{1}{2}(-28) = 12 + 14 = 26 \checkmark$

55)	A	B	C	Profit
(x) football	4x	2x		£3x
(y) basketbal	6y	6y	1y	£2y
total	120	72	10	

$$4x + 6y \leq 120$$

$$2x + 6y \leq 72$$

$$y \leq 10$$

$$x \geq 0$$

$$y \geq 0$$

$$\begin{array}{r} x/y \\ 0/20 \\ 30/0 \end{array}$$

$$\begin{array}{r} x/y \\ 0/12 \\ 36/0 \end{array}$$

$$\text{Profit} = 3x + 2y$$

- $(0, 0) \Rightarrow 3(0) + 2(0) = 0$
- $(0, 10) \Rightarrow 3(0) + 2(10) = 20$
- $(6, 10) \Rightarrow 3(6) + 2(10) = 38$
- $(24, 4) \Rightarrow 3(24) + 2(4) = 80$
- $(30, 0) \Rightarrow 3(30) + 2(0) = 90$

make 30 footballs & 0 basketballs

